A MATHEMATICAL APPROACH: SPRINKLER IRRIGATION DROP DISTRIBUTION ON SOIL SURPACE

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Abstract

The objectives of this study were to develop an equation and derive equation for different boundary conditions of sprinkler irrigation drop falling and to stop runoff from soil surface. Irrigation scheduling is the process related to when, how much water to apply to a soil. The Irrigation method concerns "how" that desired water depth is applied to the field. The uniformity of water distribution depends on an irrigated field and efficiency of on-farm water application. Conclusions point out on the laterals must install parallel to field slope contours for controlling runoff, erosion and on-farm water application.

Key words: mathematical; model; drop; distribution

INTRODUCTION

Irrigation scheduling requires knowledge on crop water requirements (evapotranspiration) and yield responses to water, the constraints specific to each irrigation method and irrigation equipment, the limitation relative to the water supply system and the financial and economic implications of the irrigation practice (Heermann, 1996). To improve the irrigation method requires the consideration of the factor influencing the hydraulic processes, the water infiltration and the uniformity of water application to entire field (Hlavek, 1992). The consideration of all these aspects makes irrigation management a complex decision making and field practice process (Pereira, 1999).

Sprinkler irrigation is suitable for most crops. It is also adaptable to nearly all irrigable soils, because sprinklers are available for a wide range of discharge capacities. Where soils have low water-holding capacity and shallow-rooted crops are to be irrigated, lighter and more frequent irrigations are required. Fixed and continuously moving systems are both suitable for such applications. The flexibility of present sprinkler equipment and its efficient control of water application make the method almost universally applicable. Its usefulness for most topographic conditions is subject only to limitations imposed by land use capability and economics. It can be adapted to most climatic conditions where irrigated agriculture is practical (Keller and Bliesner, 1990; Gencoglan et al., 2005).

The sprinkler irrigation method can be operated with application rate higher than the steady state infiltration rate. In general, this type of operation is implemented for stationary sprinkler systems. The sprinkler application rate will be lower that the infiltration rate immediately after irrigation commences, and all the water applied infiltrates into the soil. As time passes, the infiltration rate decreases and becomes less then the sprinkler application rate.

When irrigation continues after this point, runoff occurs (James, 1988; James and Larson, 1976), although the amount of runoff will step depend on the amount of water that can accumulate in small surface depressions and on the slope.

This paper aims at approaching mathematical models which could contribute to the achievement of higher irrigation performances when drop distributions and surface runoff on soil.

Algebraic model formulation

In this section an algebraic model formulation develop the effective drop distribution. For this reason, precipitation of drop from sprinkler nozzle and wetted area on soil must be detected well for distribution uniformity.

In sprinkler irrigation, in the wetted area drops, wetted distribution determine as a function (D(〉)) which in symmetric to axis (Figure 1). Wetted distribution in circle shape with a r radius. If distribution boundary as given below:

$$D\rangle \quad \begin{cases} f\rangle , & \rangle\ \delta\ r \\ 0, & \rangle\ !\ r \end{cases} \qquad (1)$$

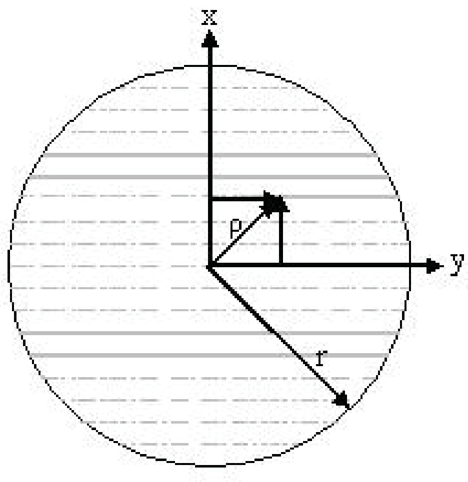

Fig.1: Under ideal condition irrigation drop distribution

Meaning, the drop from nozzle constitutes parallel and equal distance lines in wetting circle. Same of irrigation water infiltrate to soil but some of water accumulate soil surface. Accumulate water will be same quantity between two lines (band). In this case wetted distribution can be described by Eq. (2):

$$\rho(x) = \sqrt{x^2 + y^2} \qquad (2)$$

x and y is coordinates points. S(x) is the volume of water between two lines. Water volume is described by Eq. (3):

$$S(x) = 2\int_0^y D(\rho)\,dy = 2\int_0^y f(\rho)\,dy \qquad (3)$$

as it is known:

$$x^2 + y^2 = r^2 \Rightarrow y = \sqrt{r^2 - x^2} \qquad (4)$$

adding equation (4) into equation (3) resulting S(x) Eq. (5):

$$S(x) = 2\int_0^{\sqrt{r^2 - x^2}} f\left(\sqrt{x^2 + y^2}\right) dy \qquad (5)$$

Band length for unit width is $l = 2y = 2\sqrt{r^2 - x^2}$. Since irrigation distribution (g(x)) is described by Eq.

$$g(x) = \frac{S(x)}{l} = \frac{\int_0^{\sqrt{r^2 - x^2}} f\left(\sqrt{x^2 + y^2}\right) dy}{\sqrt{r^2 - x^2}} \qquad (6)$$

Irrigation distribution will work different boundary conditions. There are some scenarios to occur:

Scenario 1

Suppose that wetted drop distribution $f(\rho) = a$, g(x) is determining Eq. (7):

$$g(x) = \frac{1}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} a\,dy = \frac{a}{\sqrt{r^2 - x^2}} y\Big|_0^{\sqrt{r^2 - x^2}} = \frac{a\sqrt{r^2 - x^2}}{\sqrt{r^2 - x^2}} = a \qquad (7)$$

A result shows that drop distribution is uniform.

Scenario 2

Suppose that wetted drop distribution $f\ \rangle\quad b\rangle$, g(x) determining Eq. (8):

$$g\ x\quad \frac{\sum_{0}^{\sqrt{r^2\ x^2}} f\ \rangle\ dy}{\sqrt{r^2\quad x^2}}\quad \frac{1}{\sqrt{r^2\quad x^2}}\sum_{0}^{\sqrt{r^2\ x^2}} b\rangle dy\quad \frac{b}{\sqrt{r^2\quad x^2}}\sum_{0}^{\sqrt{r^2\ x^2}}\sqrt{x^2\quad y^2}\,dy$$

$$\frac{b}{\sqrt{r^2\quad x^2}}\spadesuit\frac{y}{2}\sqrt{x^2\quad y^2}\quad\frac{x^2}{2}\ln y\quad\sqrt{x^2\quad y^2}\ \equiv\approx\cdots\Big|_{0}^{\sqrt{r^2\ x^2}}\quad\frac{b}{\sqrt{r^2\quad x^2}}\clubsuit\frac{r\sqrt{r^2\quad x^2}}{2}\quad\frac{x^2}{2}\ln\frac{\sqrt{r^2\quad x^2}\quad r}{x}\bullet\div\neq$$

$$\frac{br}{2}\quad\frac{bx^2}{2\sqrt{r^2\quad x^2}}\ln\frac{\sqrt{r^2\quad x^2}\quad r}{x} \tag{8}$$

Scenario 3

Suppose that wetted drop distribution $f\ \rangle\quad c\rangle^2$,
g(x) determining Eq. (9):

$$g\ x\quad\frac{1}{\sqrt{r^2\quad x^2}}\sum_{0}^{\sqrt{r^2\ x^2}} c\ \sqrt{x^2\quad y^2}^{\,2}\ dy\quad\frac{c}{\sqrt{r^2\quad x^2}}\sum_{0}^{\sqrt{r^2\ x^2}} x^2\quad y^2\ dy$$

$$\frac{c}{\sqrt{r^2\quad x^2}}\ x^2y\quad\frac{y^3}{3}\Big)\Big|_{0}^{\sqrt{r^2\ x^2}}\quad\frac{c}{\sqrt{r^2\quad x^2}}\spadesuit x^2\sqrt{r^2\quad x^2}\quad\frac{\sqrt{r^2\quad x^2}^{\,3}}{3}\equiv\approx\approx\cdots$$

$$\frac{c}{\sqrt{r^2\quad x^2}}\spadesuit x^2\sqrt{r^2\quad x^2}\quad\frac{r^2\quad x^2\sqrt{r^2\quad x^2}}{3}\equiv\approx\approx\cdots\quad c\clubsuit x^2\quad\frac{r^2\quad x^2}{3}\bullet\div\neq\quad\frac{c(2x^2\quad r^2)}{3} \tag{9}$$

Scenario 4

Suppose that wetted drop distribution is parabolic $f\ \rangle\quad a\quad b\rangle\quad c\rangle^2$, g(x) determining Eq. (10):

$$g\ x\quad\frac{1}{\sqrt{r^2\quad x^2}}\sum_{0}^{\sqrt{r^2\ x^2}} a\quad b\rangle\quad c\rangle^2\ dy\quad a\quad\frac{br}{2}\quad\frac{bx^2}{2\sqrt{r^2\quad x^2}}\ln\frac{\sqrt{r^2\quad x^2}\quad r}{x}\quad\frac{c(2x^2\quad r^2)}{3} \tag{10}$$

The result of Eq. (10) is meaning irrigation drops distribute as equal to soil surface.

Lets look at interaction between irrigation water and soil surface while irrigation nozzle circling (Figure 2).

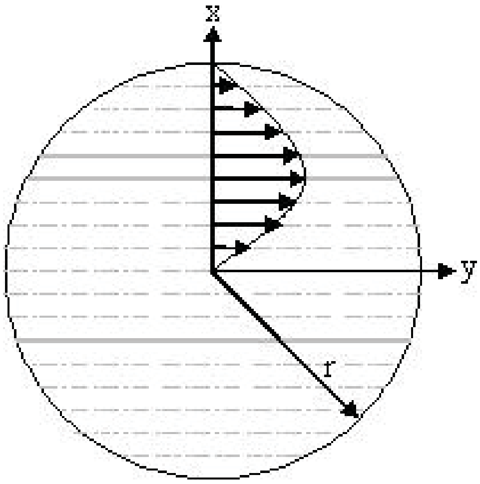

Fig. 2: Irrigation drop distribution with sprinkle nozzle moving on y axis

For this case boundary conditions given Eq. (11):

$$D \rangle \quad \begin{cases} f \rangle, & \rangle \tau r \\ 0, & \rangle \quad r \end{cases} \qquad (11)$$

In this case, water source point moves from y axis. Since S(x) (volume of water) be formed small water volumes on coordinates x and y, thus can be shown as Eq. (12):

$$g_k \ x \quad 2\int_0^{y} f \ \rangle \ dy \quad 2\int_0^{\sqrt{r^2 \ x^2}} f \sqrt{x^2 \quad y^2} \ dy \qquad (12)$$

There are some other cases that in equation (11) boundary condition.

Scenario 5

Suppose that wetted drop distribution is :

$f \ \rangle \quad a \quad b\rangle \quad c\rangle^2$ result is Eq. (13)

with using equation (12)

$$g_{kk} \ x \quad 2\int_0^{\sqrt{r^2 \ x^2}} a \ b\rangle \quad c\rangle^2 \ dy \quad (2a \ br)\sqrt{r^2 \ x^2} \quad bx^2 \ln\frac{\sqrt{r^2 \ x^2} \ r}{x} \quad \frac{(4cx^2 \ 2cr^2)\sqrt{r^2 \ x^2}}{3} \qquad (13)$$

Scenario 6

Suppose starting drop distribution $D\rangle$ as an n degree polynomial, it occurs such as $f\rangle \quad \sum_{i\,0}^{n} a_i \rangle_i$ when we look at the first four term equation of the equation $(f\rangle \quad a_0 \quad a_1\rangle \quad a_2\rangle^2 \quad a_3\rangle^3 \quad a_4\rangle^4 \quad \ldots)$ can be written as Eq. (14, 15)). According to case results irrigation distribution function depends on x and r values.

$$g_{kp}\ x \quad 2 \sum_{0}^{\sqrt{r^2 \ x^2}} a_0 \quad a_1\rangle \quad a_2\rangle^2 \quad a_3\rangle^3 \quad a_4\rangle^4 \ \ldots\ dy \quad (2a_0 \quad a_1 r)\sqrt{r^2 \quad x^2} \quad a_1 x^2 \ln\frac{\sqrt{r^2 \quad x^2} \quad r}{x}$$

$$\frac{(4a_2x^2 \quad 2a_2r^2)\sqrt{r^2 \quad x^2}}{3} \quad 2a_3\left(\frac{r(2r^2 \quad 3x^2)\sqrt{r^2 \quad x^2} \quad 3x^4 \ln\frac{\sqrt{r^2 \quad x^2} \quad r}{x}}{8}\right) \ldots$$

$$2a_4\left(\frac{(8x^4 \quad 4x^2r^2 \quad 3r^4)\sqrt{r^2 \quad x^2}}{15}\right) \quad \ldots \qquad (14)$$

$$\text{or } g_{kp}\ x \quad \sum_{0}^{\sqrt{r^2 \ x^2}} a_0 \quad a_1\rangle \quad a_2\rangle^2 \quad a_3\rangle^3 \quad a_4\rangle^4 \ \ldots\ dy \quad g_{kk}\ x_{\ a\ a_0;b\ a_1;c\ a_2}$$

$$2a_3\left(\frac{r(2r^2 \quad 3x^2)\sqrt{r^2 \quad x^2} \quad 3x^4 \ln\frac{\sqrt{r^2 \quad x^2} \quad r}{x}}{8}\right) \quad 2a_4\left(\frac{(8x^4 \quad 4x^2r^2 \quad 3r^4)\sqrt{r^2 \quad x^2}}{15}\right) \quad \ldots \qquad (15)$$

RESULTS AND DISCUSSIONS

The cases show that in different boundary conditions with different scenarios. When the field separates equal apart lines (band) movement of sprinkler irrigation nozzle has to be y axis. Derivations show that starting drop distribution is not dependent and irrigation distribution (g(x) = constant) is become a constant.

Irrigating in cropping season soil surface preparation and laterals must install parallel to slope contours.

CONCLUSIONS

Irrigation distribution in different boundary conditions were discussed with an algebraic model formulation, the main scenario was when infiltration becomes constant surface flow starts. As it is known infiltration does not stops but it works with surface flow together. There must be some other scenarios to get more accurate results such as field slope and precipitations (rain).

As results shows that another conclusion is that before cropping and irrigation to decrease surface flow field must till parallel to contours.

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